

Multiple M2-branes and plane waves

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ABSTRACT: We propose a natural generalisation of the BLG multiple M2-brane action to membranes in curved plane wave backgrounds, and verify in two different ways that the action correctly captures the non-trivial space-time geometry. We show that the M2 to D2 reduction of the theory along a non-trivial direction in field space is equivalent to the D2-brane world-volume Yang-Mills theory with a non-trivial (null-time dependent) dilaton in the corresponding IIA background geometry. As another consistency check of this proposal we show that the properties of metric 3-algebras ensure the equivalence of the Rosen coordinate version of this action (time-dependent metric on the space of 3-algebra valued scalar fields, no mass terms) and its Brinkmann counterpart (constant couplings but time-dependent mass terms). We also establish an analogous result for deformed Yang-Mills theories in any dimension which, in particular, demonstrates the equivalence of the Rosen and Brinkmann forms of the plane wave matrix string action.

KEYWORDS: M-Theory, p-branes, Penrose limit and pp-wave background.

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1. Introduction

The recent Bagger-Lambert-Gustavsson (BLG) proposal for a world-volume theory of multiple membranes [1, 2], following earlier work [3], in terms of a 3-algebra gauge theory has already received considerable attention. Various properties of the BLG theory have been analysed e.g. in [4–6], and a generalisation of the BLG theory to Lorentzian 3-algebras associated to ordinary Lie algebras has been proposed in [7]. The role of the 3-algebra structure for 1-loop corrections to the BLG theory has been discussed in [8].

At the moment it is not completely clear [9, 10] if the Lorentzian 3-algebras really give a theory of multiple uncompactified membranes in 11 dimensions or if they just provide an exotic rewriting of the D2-brane world-volume theory [11, 12], and an alternative generalisation of the BLG theory has been proposed in [13].

Nevertheless, deformations of the (generalised) BLG theories [1, 2, 7] may provide a Lagrangian description of multiple M2-branes in non-trivial backgrounds and may also, in any case, be of interest in their own right. Given the scarcity and rigidity of finite-dimensional Euclidean [14] and Lorentzian [15] 3-algebras, one has to look elsewhere for suitable modifications. Certain mass [16] and Janus-like [17] deformations have already been considered, other variations of the BLG action are discussed in [18], and a unified description of various deformations of the BLG theory has been given in [19].

In this same spirit, but along somewhat different lines, we propose that the 3-algebra action with scalar sector

$$\begin{aligned}
 S_{RC-BLG}[X^I] = \int d^n \sigma \operatorname{Tr} & \left(-\frac{1}{2} g_{IJ}(t) (D_\alpha X^I, D^\alpha X^J) \right. \\
 & \left. - \frac{1}{2 \cdot 3!} g_{IL}(t) g_{JM}(t) g_{KN}(t) ([X^I, X^J, X^K], [X^L, X^M, X^N]) \right)
 \end{aligned}
 \tag{1.1}$$

describes (for $n = 3$) multiple membranes extended along the (x^\pm, x^9) -directions in the general Rosen coordinate (RC) plane wave background

$$ds^2 = 2dx^+ dx^- + (dx^9)^2 + \sum_{I,J=1}^8 g_{IJ}(x^+) dx^I dx^J , \quad (1.2)$$

in the same way that the BLG action (to which it reduces for $g_{IJ}(t) = \delta_{IJ}$) describes membranes in flat space (or some suitable M-orbifold thereof [6]).¹

In the absence of any straightforwardly applicable symmetry considerations (the above Lagrangian will in general have no global symmetries, and the total action, with fermions, is not expected to have any linearly realised supersymmetries, since plane wave backgrounds are generically 1/2 BPS), we will perform two other consistency checks on this proposal which show that the action (1.1) correctly captures the plane wave space-time geometry.

First (section 3) we consider the analogue of the M2 to D2 [4] reduction procedure for the Lorentzian 3-algebras [7] (perhaps more appropriately referred to as D2 to D2 [11]) in the presence of a non-trivial metric component $g_{88}(x^+)$ along the direction X^8 in field space that is being vev'ed. We show that the resulting 2+1 dimensional Yang-Mills theory has an effective time-dependent Yang-Mills coupling constant

$$g_{YM}^2(t) = g_{88}(t)g_{YM}^2 , \quad (1.3)$$

and that this is identical to the coupling constant one finds from the world-volume theory of multiple D2-branes in the presence of a non-trivial dilaton (with x^8 considered as the compactified M-theory direction).

Another consistency check is provided by the observation (section 4) that the action (1.1) is related to the apparently completely different 3-algebra action

$$S_{BC-BLG}[Z^A] = \int d^n \sigma \text{Tr} \left(-\frac{1}{2} \delta_{AB} (D_\alpha Z^A, D^\alpha Z^B) + \frac{1}{2} A_{AB}(t) (Z^A, Z^B) - \frac{1}{2.3!} \delta_{AD} \delta_{BE} \delta_{CF} ([Z^A, Z^B, Z^C], [Z^D, Z^E, Z^F]) \right) \quad (1.4)$$

(no time-dependent couplings on the scalar field space but arbitrary time-dependent mass terms instead, encoded in the matrix $A_{AB}(t)$) by a simple linear transformation of the fields,

$$S_{RC-BLG}[X^I = E^I_A Z^A] = S_{BC-BLG}[Z^A] . \quad (1.5)$$

The validity of (1.5) provides strong evidence that (1.1) and (1.4) encode the plane wave geometry (1.2), since it should be regarded as the 3-algebra field-theory counterpart of the statement that a plane wave can also be written in the more common Brinkmann coordinates (BC) z^μ with, in particular, $x^I = E^I_A z^A$ (4.4) as (suppressing the trivial x^9 -direction)

$$2dx^+ dx^- + g_{IJ}(x^+) dx^I dx^J = 2dz^+ dz^- + A_{AB}(z^+) z^A z^B (dz^+)^2 + \delta_{AB} dz^A dz^B . \quad (1.6)$$

¹See [20] for a complementary discussion of non-trivial backgrounds from the M5-brane Nambu-Goto action point of view.

Note that in these coordinates, the membrane is stretched along the metrically non-trivial (z^\pm, z^9) -directions. Thus the dependence of the induced world-volume metric on the transverse coordinates via the quadratic $A_{AB}z^Az^B$ -terms manifests itself through mass terms in the action (1.4), as in the case of strings in the lightcone gauge. This also provides us with a geometric interpretation of an arbitrary mass deformation of the BLG theory in terms of plane waves (in the absence of fluxes, we should also require the 11d vacuum Einstein equations to be satisfied, namely that A_{AB} be traceless).

Note also that the BC form of the action (1.4) explains why we focus on plane wave space-times here (since in principle we could have e.g. allowed the couplings g_{IJ} in (1.1) to depend on all the world-volume coordinates). First of all, the analogy with the quantisation of strings in the lightcone gauge suggests that the BLG action is itself a lightcone gauge fixed action. Such a gauge fixing is typically still possible e.g. for more general pp-wave backgrounds in which the wave profile $A_{AB}(z^+, z^A)$ is not quadratic, but in that case we would have to address the issue of how to define $\text{Tr}(Z_1^A, \dots, Z_k^A)$ for $k \neq 2$, while the quadratic mass term in (1.4) is unambiguous. For the same reason we would also not want to consider a dependence of g_{IJ} in (1.2) or (1.1) on the transverse coordinates x^I or scalars X^I .

The 3-algebra gauge invariance of the actions, in particular the existence of the invariant scalar product $\text{Tr}(\cdot)$, turns out to play a crucial role in the proof of (1.5). Along the way, we will also establish an analogous result for Yang-Mills theories, which implies in particular the equivalence of the Rosen and Brinkmann versions of the matrix string action for plane waves [21].²

2. Plane wave Yang-Mills and 3-algebra actions

The scalar sector of a prototypical non-Abelian Yang-Mills + scalar action in n space-time dimensions (with the flat world-volume metric $\eta_{\alpha\beta}$) has the form

$$S_{\text{YM}} = \int d^n\sigma \text{Tr} \left(-\frac{1}{2} \delta_{IJ} D_\alpha X^I D^\alpha X^J - \frac{1}{4} g_{\text{YM}}^2 \delta_{IK} \delta_{JL} [X^I, X^J] [X^K, X^L] \right) . \quad (2.1)$$

To bring out the analogies with, and differences to, the 3-algebra actions, we recall here that the $X^I = X_a^I T^a$ are adjoint (Lie algebra valued) scalar fields, $[T^a, T^b] = f^{ab}{}_c T^c$, Tr is an invariant scalar product (under the ad-action $[T^a, \cdot]$, which acts as a derivation of the Lie bracket - the Jacobi identity) on the Lie algebra,

$$\text{Tr}[T^a, T^b]T^c + \text{Tr} T^b[T^a, T^c] = 0 , \quad (2.2)$$

and the covariant derivative is $D_\alpha X_a^I = \partial_\alpha X_a^I - f^{bc}{}_a A_{\alpha b} X_c^I$.

²In the spirit of the CSV matrix big bang model [22], these provide a non-perturbative description of string theory in a plane wave background - see [21] for details and further references, since matrix string theory is not our main concern in this short note.

Likewise the scalar sector of a prototypical 3-algebra action, namely the BLG action [1, 2] (now blindly generalised to n dimensions), is

$$S_{BLG} = \int d^n \sigma \operatorname{Tr} \left(-\frac{1}{2} \delta_{IJ} (D_\alpha X^I, D^\alpha X^J) - \frac{1}{2 \cdot 3!} \delta_{IL} \delta_{JM} \delta_{KN} ([X^I, X^J, X^K], [X^L, X^M, X^N]) \right). \quad (2.3)$$

Here the $X^I = X_a^I T^a$ are 3-algebra valued scalar fields,

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d, \quad (2.4)$$

$\operatorname{Tr}(\ , \)$ is an invariant scalar product (under the action of $[T^a, T^b, \]$, which acts as a derivation of the 3-bracket - the ‘fundamental identity’) on the 3-algebra,

$$\operatorname{Tr}([T^a, T^b, T^c], T^d) + \operatorname{Tr}(T^c, [T^a, T^b, T^d]) = 0, \quad (2.5)$$

and the covariant derivative is $D_\alpha X_a^I = \partial_\alpha X_a^I - f^{bcd}{}_a A_{\alpha bc} X_d^I$.

These two basic classes of actions can be deformed in various ways, e.g. by modifying the couplings of the scalar fields, and we will consider two such modifications. The first class of actions arises from (2.1) or (2.3) by replacing the flat metric δ_{IJ} on the scalar field space by a time-dependent matrix $g_{IJ}(t)$ of ‘coupling constants’. Thus the deformed action is (suppressing the coupling constant g_{YM}^2)

$$S_{RC-YM} = \int d^n \sigma \operatorname{Tr} \left(-\frac{1}{2} g_{IJ}(t) D_\alpha X^I D^\alpha X^J - \frac{1}{4} g_{IK}(t) g_{JL}(t) [X^I, X^J] [X^K, X^L] \right) \quad (2.6)$$

in the Yang-Mills case, and (1.1) in the BLG case. The second modification consists of simply adding (possibly time-dependent) mass terms for the scalars. Thus, denoting the (same number of) scalars in this model by Z^A , the actions we will consider are

$$S_{BC-YM} = \int d^n \sigma \operatorname{Tr} \left(-\frac{1}{2} \delta_{AB} D_\alpha Z^A D^\alpha Z^B - \frac{1}{4} \delta_{AC} \delta_{BD} [Z^A, Z^B] [Z^C, Z^D] + \frac{1}{2} A_{AB}(t) Z^A Z^B \right) \quad (2.7)$$

and its 3-algebra counterpart (1.4), with $A_{AB}(t)$ minus the mass-squared matrix.

The labels RC and BC refer to the Rosen and Brinkmann coordinates of plane wave metrics, as will become clear in section 4, and for this reason we will also refer to the above actions and their 3-algebra counterparts as plane wave actions.

3. M2 (or D2) to D2 with a non-trivial dilaton

We consider the case where the metric (1.2) is of the form

$$ds^2 = 2dx^+ dx^- + (dx^9)^2 + \sum_{i,j=1}^7 g_{ij}(x^+) dx^i dx^j + g_{88}(x^+) (dx^8)^2, \quad (3.1)$$

and for the purposes of this section we may as well take $g_{ij} = \delta_{ij}$, since we just want to keep track of the effect of a non-trivial g_{88} in the M2 to D2 reduction [4, 7, 11].

We will also specifically consider the case of a Lorentzian 3-algebra [7], with generators $\{T^a\} = \{T^+, T^-, T^m\}$ and non-trivial structure constants $f^{+mnp} = 2f^{mnp}$, $f_{-mnp} = f_{mnp}$. Expanding the 3-algebra valued fields in the above basis, $X^I = X_a^I T^a$, one finds that, as in [7], the field $X_-^I \equiv X^{+I}$ appears in the action (1.1) only via the term

$$\mathcal{L}_{X^+} = -\frac{1}{2}g_{IJ}(t)\partial_\alpha X^{+I}\partial^\alpha X^{-J} + \dots, \tag{3.2}$$

leading to the equations of motion $\partial_\alpha(g_{IJ}(t)\partial^\alpha X^{-J}) = 0$. A particular solution of this equation is $X^{-8} = \text{const} \neq 0$ and $X^{-i} = 0$. Note, however, that there are other, non-constant, solutions to this equation, even when $g_{IJ}(t) = \delta_{IJ}$, employed e.g. in [17], and that even for a constant solution here we cannot appeal to $\text{SO}(8)$ -invariance to rotate such a solution into the X^8 -direction. We are thus making the specific choice of singling out this direction (corresponding to a specific M→IIA reduction), and identify the vev of X^{-8} with the Yang-Mills coupling constant, $\langle X^{-8} \rangle = g_{\text{YM}}$.

It is now easy to see, by following the procedure in [7], that the gauge invariant scalar kinetic term for X_m^8 and the BF-term of the action,

$$\begin{aligned} \mathcal{L}_B &= -\frac{1}{2}g_{88}(t)D_\alpha X_m^8 D^\alpha X_m^8 + 2\epsilon^{\alpha\beta\gamma} B_\alpha^a F_{\beta\gamma}^a + \dots \\ D_\alpha X_m^I &= \partial_\alpha X_m^I - 2B_{\alpha m} X^{-I} + f_{mnp} A_\alpha^n X^{pI} \end{aligned} \tag{3.3}$$

combine to give rise to a Yang-Mills action

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4g_{\text{YM}}^2(t)} F_{\alpha\beta}^m F^{m\alpha\beta} + \dots \tag{3.4}$$

with the time-dependent coupling constant

$$g_{\text{YM}}^2(t) = g_{88}(t)g_{\text{YM}}^2. \tag{3.5}$$

This same combination also arises from the sextic potential as the coefficient of the quartic potential term for the remaining 7 scalar fields $X^i = X_m^i T^m$, and thus we can indeed identify it as the time-dependent coupling constant of the resulting Yang-Mills theory.

How does this compare with the expectation that, somehow [9], this procedure of giving a vev to a scalar should correspond [4] to compactifying M-theory on a circle down to IIA? Since the standard relation

$$ds^2 = e^{-2\phi/3} ds_{st}^2 + e^{4\phi/3} (dx^8)^2 \tag{3.6}$$

between the M-theory and IIA string frame backgrounds implies that $g_{88} = \exp 4\phi/3$, while the YM coupling constant of the D2-brane theory is usually set by $g_{\text{YM}}^2 = g_s/\ell_s$, on the face of it this looks incompatible with (3.5). However, we have to remember that in the string frame metric

$$ds_{st}^2 = e^{2\phi/3} (2dx^+ dx^- + (dx^9)^2 + g_{ij}(x^+) dx^i dx^j) \tag{3.7}$$

the induced metric $h_{\alpha\beta}$ on the D2-brane world-volume is non-trivial. Thus the D2-brane Yang-Mills action has the form

$$\frac{1}{g_s \ell_s^3} \int d^3\sigma e^{-\phi} \sqrt{-\det h} h^{\alpha\gamma} h^{\beta\delta} \ell_s^4 F_{\alpha\beta}^m F_{\gamma\delta}^m = \frac{\ell_s}{g_s} \int d^3\sigma e^{-4\phi/3} \delta^{\alpha\gamma} \delta^{\beta\delta} F_{\alpha\beta}^m F_{\gamma\delta}^m, \quad (3.8)$$

from which we read off the coupling constant

$$g_{\text{YM}}^2(t) = (g_s/\ell_s) e^{4\phi(t)/3} \quad (3.9)$$

(in the lightcone gauge $x^+ = t$). This agrees precisely with the result (3.5) obtained from ‘Higgsing’ the Lorentizan BLG action.

4. Rosen vs Brinkmann form of plane wave actions

The purpose of this section is to establish that the two, apparently rather different, classes of Yang-Mills and 3-algebra actions introduced in section 2 are simply related by a linear, but time-dependent, field redefinition $X^I = E^I_A(t) Z^A$ of the scalar fields,

$$S_{RC-YM/BLG}[X^I = E^I_A Z^A] = S_{BC-YM/BLG}[Z^A]. \quad (4.1)$$

As mentioned in the Introduction and explained in [21], for the YM theories this claim originates from the equivalence of the matrix string theory description of plane wave backgrounds in Rosen and Brinkmann coordinates (1.6), and (4.1) is the generalisation of this assertion to arbitrary dimension n , any number of scalar fields, and to 3-algebra actions.

We could straightaway prove (4.1) by a brute-force calculation, but this would be rather unenlightening. Instead, we will first consider a much simpler classical mechanics toy model of this equivalence. We will then readily be able to establish the result for the plane wave Yang-Mills actions (2.6), (2.7). From this argument we then also learn how to use 3-algebra identities to prove (4.1) in that case.

4.1 A classical mechanics toy model

Consider the Lagrangian L_{bc} corresponding to the lightcone Hamiltonian of a particle in a plane wave in Brinkmann coordinates (in the lightcone gauge $z^+ = t$),

$$L_{bc}(z) = \frac{1}{2} (\delta_{AB} \dot{z}^A \dot{z}^B + A_{AB}(t) z^A z^B), \quad (4.2)$$

and the corresponding Lagrangian in Rosen coordinates,

$$L_{rc}(x) = \frac{1}{2} g_{IJ}(t) \dot{x}^I \dot{x}^J. \quad (4.3)$$

The claim is that these two Lagrangians are equal up to a total time-derivative. To see this, recall that the coordinate transformation between the Rosen and Brinkmann forms (1.6) of a plane wave metric has the form

$$(x^+, x^-, x^I) = (z^+, z^- + \frac{1}{2} \dot{E}_{AI} E^I_B z^A z^B, E^I_A z^A) \quad (4.4)$$

where $E^I_A = E^I_A(x^+)$ is a vielbein for $g_{IJ}(x^+)$ satisfying the symmetry condition

$$\dot{E}_{AI}E^I_B = \dot{E}_{BI}E^I_A \quad , \quad (4.5)$$

and the relation between $g_{IJ}(x^+)$ and $A_{AB}(z^+)$ can be compactly written as [23]

$$A_{AB}(z^+) = \ddot{E}_{AI}(z^+)E^I_B(z^+) \quad . \quad (4.6)$$

The symmetry condition (4.5), which can be geometrically interpreted as the statement that the frame E^I_A is parallel transported along ∂_{x^+} [24], will play a crucial role on several occasions in the following.

Substituting $x^I = E^I_A z^A$ in L_{rc} , one can now verify that one indeed obtains L_{bc} up to a total time-derivative. The way to see this without any calculation is to start with the complete geodesic Lagrangian in Brinkmann or Rosen coordinates,

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}^{(rc)}\dot{x}^\mu\dot{x}^\nu = \frac{1}{2}g_{\mu\nu}^{(bc)}\dot{z}^\mu\dot{z}^\nu \quad (4.7)$$

in the lightcone gauge $z^+ = x^+ = t$, leading to

$$L_{bc}(z) + \dot{z}^- = L_{rc}(x) + \dot{x}^- \quad . \quad (4.8)$$

This makes it manifest that the two Lagrangians $L_{bc}(z)$ and $L_{rc}(x)$ differ only by a total time-derivative, namely the derivative of the shift of x^- in the coordinate transformation (4.4).

A concrete illustration of this is provided by the standard harmonic oscillator Lagrangian

$$L_{bc}(z) = \frac{1}{2}(\dot{z}^2 - \omega^2 z^2) \quad , \quad (4.9)$$

whose equivalence with the somewhat more exotic Lagrangian

$$L_{rc}(x) = \frac{1}{2}\sin^2\omega t \dot{x}^2 \quad (4.10)$$

with a time-dependent kinetic term can be traced back to the two equivalent representations

$$2dx^+dx^- + \sin^2\omega x^+(dx)^2 = 2dz^+dz^- - \omega^2 z^2(dz^+)^2 + (dz)^2 \quad (4.11)$$

of the corresponding plane wave geometry.

4.2 Rosen to Brinkmann for Yang-Mills actions

We can now come back to the two types of Yang-Mills actions (2.6) and (2.7), which are obviously in some sense non-Abelian counterparts of the classical mechanics Brinkmann and Rosen coordinate actions $S_{bc} = \int L_{bc}$ and $S_{rc} = \int L_{rc}$ discussed above. We are thus led to consider the linear transformation

$$X^I(\sigma^\alpha) = E^I_A(t)Z^A(\sigma^\alpha) \quad (4.12)$$

of the scalar fields, where $E^I_A(t)$ is a vielbein for the metric (couplings) $g_{IJ}(t)$ on the scalar field space satisfying (4.5).

Even though in general non-Abelian coordinate transformations are a tricky issue, this particular transformation is easy to deal with since it is linear as well as diagonal in the Lie algebra. Consider e.g. the quartic potential terms in (2.6) and (2.7). With the substitution (4.12), one obviously has

$$g_{IK}g_{JL}[X^I, X^J][X^K, X^L] = g_{IK}g_{JL}E_A^I E_B^J E_C^K E_D^L [Z^A, Z^B][Z^C, Z^D] \\ = \delta_{AC}\delta_{BD}[Z^A, Z^B][Z^C, Z^D] , \quad (4.13)$$

so that the two quartic terms are indeed directly related by (4.12). Now consider the gauge covariant kinetic term for the scalars in (2.6). Since $E_A^I = E_A^I(t)$ depends only on t , the spatial covariant derivatives transform as

$$\alpha \neq t : \quad D_\alpha X^I = E_A^I(t)D_\alpha Z^A , \quad (4.14)$$

so that the spatial derivative parts of the scalar kinetic terms are mapped into each other. It thus remains to discuss the term $\text{Tr} g_{IJ}(t)D_t X^I D_t X^J$ involving the covariant time-derivatives. The term with two gauge fields A is purely algebraic and is thus mapped directly to its BC counterpart in the term $\text{Tr} \delta_{AB} D_t Z^A D_t Z^B$. For the term quadratic in the ordinary t -derivatives, the argument is identical to that in section 2.2, and thus, using (4.5) and (4.6), one finds

$$\frac{1}{2} \text{Tr} g_{IJ}(t)\dot{X}^I \dot{X}^J = \frac{1}{2} \text{Tr}(\delta_{AB}\dot{Z}^A \dot{Z}^B + A_{AB}(t)Z^A Z^B) + \frac{d}{dt}(\dots) . \quad (4.15)$$

The only remaining subtlety are terms involving the t -derivative \dot{E}_A^I of E_A^I , arising from cross-terms like

$$\text{Tr} g_{IJ}(t)[A_t, X^I]\partial_t X^J = \text{Tr} g_{IJ}(t)E_A^I [A_t, Z^A]\partial_t (E_B^J Z^B) . \quad (4.16)$$

However, these terms do not contribute at all since

$$g_{IJ}(t)E_A^I \dot{E}_B^J \text{Tr}[A_t, Z^A]Z^B = g_{IJ}(t)E_A^I \dot{E}_B^J \text{Tr} A_t [Z^A, Z^B] = 0 \quad (4.17)$$

by the ad-invariance of the trace (2.2) and the symmetry condition (4.5). This establishes (4.1).

It is pleasing to see that this symmetry condition, which already ensured several cancellations in the standard transformation from Rosen to Brinkmann coordinates (and thus also in establishing e.g. (4.15)), is also responsible for the elimination of some terms of genuinely non-Abelian origin (something the symmetry condition was not originally designed for).

The above equivalence is also valid for models with a time-dependent dilaton/coupling constant, as in [21], since the total time-derivative arises only from the (dilaton-independent) scalar kinetic term. In [21], we illustrate the advantages of the BC representation (the scalar kinetic term has the canonical form and the mass term encodes invariant geometric information about the plane wave since $A_{AB}(z^+)$ is its curvature tensor) *vis-à-vis* its RC counterpart in the matrix string context.

This equivalence also extends in a rather obvious way to the appropriate fermionic terms of the action. In these models the couplings between the fermions Ψ and the scalar fields X^I universally have the form

$$S_\Psi \sim \int d^n \sigma \text{Tr} \bar{\Psi} \Gamma_I [X^I, \Psi] . \quad (4.18)$$

Thus the only effect of the transformation (4.12) is to convert the RC gamma-matrices Γ_I to their BC (frame component) counterparts $\Gamma_A = E_A^I \Gamma_I$,

$$\Gamma_I X^I = \Gamma_A Z^A , \quad (4.19)$$

with

$$\{\Gamma_I, \Gamma_K\} = 2g_{IK} \quad \Rightarrow \quad \{\Gamma_A, \Gamma_B\} = 2\delta_{AB} . \quad (4.20)$$

4.3 Rosen to Brinkmann for 3-algebra actions

We now consider the effect of the transformation $X^I = E_A^I(t) Z^A$ of the 3-algebra valued fields on the RC action (1.1). It is straightforward to see that, exactly as in (4.13), the sextic potential term is mapped to that of the BC action (1.4), and that the YM-theory identities (4.14) and (4.15) remain valid in the 3-algebra context, so that in particular the mass terms of (1.4) are generated in this way. It thus only remains to discuss, similarly to (4.16), the cross-terms between a t -derivative and the 3-algebra connection. These have the form

$$\text{Tr}(T^a, T^c) (\partial_t X_a^I) f^{deb} {}_c \mathcal{A}_{tde} X_b^J g_{IJ}(t) = \text{Tr}(T^a, [T^d, T^e, T^b]) (\partial_t X_a^I) \mathcal{A}_{tde} X_b^J g_{IJ}(t) , \quad (4.21)$$

where we used the 3-algebra relation (2.4) in the form $f^{deb} T^c = [T^d, T^e, T^b]$. Inserting the field transformation, we now find that there is just one troublesome term, namely the one involving the t -derivative of the transformation matrix $E_A^I(t)$ itself. However, here again the symmetry condition (4.5) and the invariance of the trace (2.5) come to the rescue to show that this term is identically zero,

$$\text{Tr}(T^a, [T^d, T^e, T^b]) \dot{E}_A^I Z_a^A \mathcal{A}_{tde} E_B^J Z_b^B g_{IJ}(t) = 0 , \quad (4.22)$$

since $\text{Tr}(T^a, [T^d, T^e, T^b])$ is anti-symmetric in the indices a, b while the second part of the above expression is symmetric.

As in the YM case, the fermionic terms are also mapped to each other, since the coupling between the fermions Ψ (11d Majorana spinors subject to the constraint $\Gamma_{012} \Psi = -\Psi$) and the scalar fields is purely algebraic [1, 2, 7], so that

$$\text{Tr}(\bar{\Psi}, \Gamma_{IJ} [X^I, X^J, \Psi]) = \text{Tr}(\bar{\Psi}, \Gamma_{AB} [Z^A, Z^B, \Psi]) . \quad (4.23)$$

The results of this and the previous section provide us with reasonable (albeit still rather circumstantial) evidence that the deformed BLG actions (1.1), (1.4) that we have proposed indeed describe multiple M2-branes in a curved plane wave background, but much remains to be understood regarding the BLG actions, their extension to curved space-times, and their relation to multiple M2-branes in general.

Acknowledgments

We are grateful to Denis Frank, Giuseppe Milanesi and Sebastian Weiss for discussions and for their collaboration on related matters. This work has been supported by the Swiss National Science Foundation and by the EU under contract MRTN-CT-2004-005104.

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